

A geometrical origin of the right-handed neutrino magnetic moment

M. Novello^{*†} and E. Bittencourt[‡]

*Instituto de Cosmologia Relatividade Astrofísica ICRA - CBPF
Rua Dr. Xavier Sigaud, 150, CEP 22290-180, Rio de Janeiro, Brazil*

(Dated: January 18, 2012)

We propose a new form of contribution for the anomalous magnetic moment of all particles. This common origin is displayed in the framework of the recent treatment of electrodynamics, called Dynamical Bridge Method, and its corresponding introduction of an electromagnetic metric which has no gravitational character. This electromagnetic metric constitutes a universal process perceived by all bodies, charged or not charged. As such it yields automatically a conclusive explanation for the existence of a magnetic moment for the right-handed neutrino.

PACS numbers: 03.50.-z, 13.40.Em, 14.60.-z, 14.60.St.

I. INTRODUCTION

In this paper we argue that the magnetic moment of any particle is compound of two very distinct parts: the standard one that depends on the charge of the particle (and consequently, from dimensional analysis, is inversely proportional to the corresponding mass); and another part (the metrical magnetic moment) that depends linearly on the mass. This second part – which is many orders of magnitude lower than the standard one, as we shall see – is common for all particles and does not depend on the charge. We shall argue that part of the anomalous magnetic moment of the electron (and others particles) is due to the metrical magnetic moment. We analyze the specific case of the neutrino to exemplify this.

Although neutrino does not have a standard magnetic moment, once it has no charge, there have been investigations concerning the possibility that beyond the standard model neutrino could have an effective magnetic one [1], [2], [3] and [4]. For any charged particle the magnetic moment (μ) is inversely proportional to the corresponding mass. However in the case of neutrino ν it has been suggested that μ_ν depends linearly on its mass. Such strange dependence is a consequence of the way the neutrino magnetic moment was introduced. The purpose of the present work is to suggest a new way to understand the origin of such magnetic moment which also depends linearly on the mass of the neutrino.

II. THE ELECTROMAGNETIC DYNAMICAL BRIDGE

In a recent work [5] it was shown that the linear Maxwell electrodynamics described in a flat Minkowski background is dynamically equivalent to the Born-Infeld theory in a curved space-time endowed with an associated

metric $\hat{q}^{\mu\nu}$. In other words, there is a Dynamical Bridge relating these two theories in such a way that they are distinct representations of one and the same dynamics. This means that *any* solution of the former will be also a solution of the latter, which is given in terms of a prescribed map. This highly nontrivial task only becomes possible if the metric of the space-time depends explicitly on the electromagnetic field. Due to the algebraic structure of the electromagnetic two-form $F_{\mu\nu}$ and its dual, there exist a kind of closure relation that allows the existence of this mapping, thus generating a “Dynamical Bridge” (DB) between the two paradigmatic theories. The price payed is to leave the Minkowski background $\eta_{\mu\nu}$ and go to a specific curved space-time $\hat{q}_{\mu\nu}$ that is constructed solely in terms of the background metric and the electromagnetic fields. We call DB the mapping that implements such a modification of representation. The complete recipe of the map and the corresponding discussion of its properties was made in the quoted paper [5]. At first sight this procedure may appears as a simple mathematical tool. However we shall use this equivalence in order to reveal new physics. Let us first of all make a short resume of the proposal of the DB.

Let a field $F_{\mu\nu}$ be defined in a flat Minkowski geometry $\eta_{\mu\nu}$ and satisfying Maxwell equation of motion. It is straightforward to show that this field satisfy the Born-Infeld dynamics, that is,

$$\partial_\nu \left(\frac{\sqrt{-\hat{q}}}{\hat{U}} (\hat{F}^{\mu\nu} - \frac{1}{4\beta^2} \hat{G} \hat{F}^{*\mu\nu}) \right) = 0, \quad (1)$$

in a curved space-time endowed with the electromagnetic metric defined as

$$\hat{q}^{\mu\nu} \equiv a \eta^{\mu\nu} + b \Phi^{\mu\nu}, \quad (2)$$

where \hat{q} is the determinant of the metric $\hat{q}_{\mu\nu}$ and

$$\Phi_{\mu\nu} \equiv F_{\mu\alpha} F^\alpha{}_\nu.$$

The coefficients a and b are given functions of the invariants $F \equiv F_{\mu\nu} F^{\mu\nu}$ and $G \equiv F^{\mu\nu} F^*_{\mu\nu}$ chosen in such a way to make both dynamics equivalent. All quantities with a hat are raised and lowered with the $\hat{q}^{\mu\nu}$ metric. In

^{*}M. Novello is Cesare Lattes ICRA Net Professor

[†]Electronic address: novello@cbpf.br

[‡]Electronic address: eduhsb@cbpf.br

particular, the invariants \hat{F} and \hat{G} are constructed with the $\hat{q}^{\mu\nu}$ metric¹.

In the case $F \ll \beta^2$, as it was shown in [5] the metric can be approximated as

$$\hat{q}_{\mu\nu} \approx \eta_{\mu\nu} + \frac{1}{2\beta^2} \phi_{\mu\nu}. \quad (3)$$

In this paper we shall examine only this case.

Henceforth we will call MM-representation when one chooses to describe electromagnetic effects in the standard Maxwell linear theory in the flat Minkowski space-time. On the other hand, when one chooses to take the dynamical bridge approach and describe the same effects in the Born-Infeld non-linear theory in a curved space-time driven by the metric $\hat{q}^{\mu\nu}$ it will be called the \hat{Q} -representation. Let us emphasize that these representations describe one and the same dynamics.

III. THE ELECTROMAGNETIC METRIC

The presence of a curved structure of the space-time in the realm of electromagnetic fields can be an important theoretical instrument of analysis only if one introduces a prescription of how matter can perceive this geometry. The question is: how matter interacts with the electromagnetic metric? In order to explore this metric formulation we have to generalize it to deal with the presence of matter. At this point we propose the natural idea that **all kind of particles, charged or not, interacts with the $\hat{q}_{\mu\nu}$ in one and same way.** The simplest mode to realize this is by using the minimal coupling principle. Let us examine the specific case of spinorial fields (e.g. the neutrino and the electron) to investigate what kind of new effects one can envisage within this metrical scenario.

This extended dynamical bridge concerns the behavior of all kind of matter. In the \hat{Q} -representation the assumption of complete democracy – that is, the idea that any kind of charged-or-not matter lives in the \hat{Q} -geometry – must have an equivalence in the Maxwell-Minkowski (MM) representation. The equation of motion of the neutrino that includes the metric effects of the electromagnetic geometry must have an equivalence in the MM-representation; one has to accept that in this representation there should exist an extra term in the Lagrangian which is the analogue of the $\hat{q}^{\mu\nu}$ -metric coupling. For instance, the role of the parameter β that appears in the \hat{Q} -representation becomes in the MM-representation the ratio between the magnetic moment of any particle and its corresponding mass, as it will be shown latter on.

IV. MINIMAL COUPLING PRINCIPLE

Let us describe electromagnetic effects in the \hat{Q} -representation. We can explore the dynamical equivalence displayed by the DB and envisage an extended way to couple any matter with the electromagnetic field. In the traditional way only charged matter is able to couple directly to the electromagnetic field. However, the modification of the metric structure due to the presence of the electromagnetic field allows a new possibility that we will explore.

The equivalence between Maxwell dynamics in a flat space-time and Born-Infeld theory was shown by considering only the free field. We extend this equivalence by assuming that matter coupling to the EM field can be described either in the MM-representation or in the $\hat{q}_{\mu\nu}$ framework. Let us investigate the consequences of the analysis in the \hat{Q} -representation. We start by noting that there are two possible forms of the interaction of matter with EM field, that is

- through the potential A_μ ;
- through the metric tensor $\hat{q}^{\mu\nu}$.

Once this second form was not considered before, we concentrate here only in the second way of interaction. Let us point out that the existence of the electromagnetic geometry has further consequences only if this geometry is perceived for all kind of matter. We are then led to state an apparently counter-intuitive assertion: every single body lives in the electromagnetic geometry $\hat{q}^{\mu\nu}$. This means that a particle can couple to the electromagnetic field without having electric charge. To be precise: if a particle has a charge it couples with the electromagnetic field through the standard channel via the potential A_μ . This happens only for the class of bodies that are named *charged*. On the other hand all particles (charged or not) interacts with the electromagnetic geometry in a unique and same way. Let us see the observational consequences of this.

V. COUPLING THE NEUTRINO TO THE ELECTROMAGNETIC FIELD

In the case of neutrino that does not have a charge, there is only the metric way to couple directly to the electromagnetic field. Let us apply the minimal coupling principle in this case.

In the electromagnetic metric we define the associated $\hat{\gamma}^\alpha$ matrices by

$$\hat{\gamma}^\mu \hat{\gamma}^\nu + \hat{\gamma}^\nu \hat{\gamma}^\mu = 2\hat{q}^{\mu\nu} \mathbf{1}, \quad (4)$$

where $\mathbf{1}$ is the identity in the associated Clifford algebra. From the equation (3) we obtain²

¹ See the appendix to find the definition of quantities that appear in this expression, like \hat{U} and others.

² The most general expression of the $\hat{\gamma}^\mu$ in terms of the constant

$$\hat{\gamma}^\mu = \gamma^\mu - \frac{1}{4\beta^2} \phi^\mu{}_\alpha \gamma^\alpha. \quad (5)$$

Therefore, the evolution equation for the spinor field becomes

$$i\hbar c \hat{\gamma}^\mu \hat{\nabla}_\mu \Psi - mc^2 \Psi = 0, \quad (6)$$

where the covariant derivative is given in terms of the Christoffel symbol constructed with the electromagnetic metric and the internal connection is provided by the Fock-Ivanenko coefficients obtained from the Riemannian condition

$$\hat{\nabla}_\mu \hat{\gamma}^\nu = [V_\mu, \hat{\gamma}^\nu], \quad (7)$$

where V_μ is an element of the associated Clifford algebra. In the absence of any kind of matter the commutator that appears in the right-hand-side vanishes. However, when matter of any kind exists then V_μ depends on both the electromagnetic field and on the properties of the matter field and it is provided by

$$V_\mu = i \frac{mc}{\hbar} \frac{F_{\mu\nu}}{\beta} \hat{\gamma}^\nu \gamma_5. \quad (8)$$

Note that (7) is the general condition that allows the geometry to be Riemannian [6], that is the covariant derivative of the metric vanishes: $\hat{\nabla}_\alpha \hat{g}^{\mu\nu} = 0$.

Using these expressions and (3) into the equation of motion of Ψ it becomes

$$i\hbar c \gamma^\mu \partial_\mu \Psi + \frac{mc^2}{\beta} F_{\mu\nu} \sigma^{\mu\nu} \gamma_5 \Psi - mc^2 \Psi = 0. \quad (9)$$

The Hermitian conjugate equation for $\bar{\Psi}$ is

$$i\hbar c \gamma^\mu \partial_\mu \bar{\Psi} - \frac{mc^2}{\beta} \bar{\Psi} F_{\mu\nu} \sigma^{\mu\nu} \gamma_5 + mc^2 \bar{\Psi} = 0, \quad (10)$$

where $\sigma^{\mu\nu} = 1/2 (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$.

We note that the net effect of the minimal coupling is to provide an effective magnetic moment for the right-handed neutrino that depends on its mass and on the parameter β , that is

$$\mu_Q = \frac{m_\nu c^2}{\beta}. \quad (11)$$

Let us point out that the hypothesis of universality of the $\hat{g}^{\mu\nu}$ revealed the existence of an extra source for

the origin of the magnetic moment of all - charged or not - particles. The compatibility of this with Maxwell-Minkowski representation implies that the Lagrangian describing the interaction of the neutrino with the electromagnetic field contains an extra term that in this MM-representation should be put by hand. In other words, the presence of a magnetic moment for the neutrino in the standard MM-representation should not be viewed as an exotic surprise but instead should be understood - on the light of the Dynamical Bridge - as a consequence of the universality of the \hat{g} geometry in the \hat{Q} -representation.

VI. COUPLING THE ELECTRON TO THE ELECTROMAGNETIC FIELD

It is important to point out here that the value of the magnetic moment obtained in the previous section contains only the general contribution for any particle, be it charged or not. Charged particles have an extra source for μ that is related to their charge. The electron standard magnetic moment, for instance, is the Bohr magneton $\mu_B = e\hbar/2m_e$. Thus the total value of the electron magnetic moment should read

$$\mu_e = \mu_B + \frac{m_e c^2}{\beta}.$$

The first part is the standard term and the second corresponds to the geometrical part. Quantum corrections could be added to this formula.

VII. COMPARISON WITH EXPERIMENTS

In this section we compare the above proposal of the contribution of the electromagnetic metric to the magnetic moment of charged and non-charged particles with observational results. We note that there are just two precise measurements for the anomalous magnetic moment. The first one is the case of the electron (the most precise one) which allows tests of the QED, in particular the fine-structure constant α . The second one is the muon magnetic moment which however is less precise but can test the entire Standard Model [8]. Nowadays, the anomalous magnetic moment of the τ is unobservable due to its very short mean life. However, there is an estimate of the anomaly that yields an upper limit of less than 13×10^{-3} . We will not analyze here the case of composed particles, like protons and neutrons, once physics of their components is still under construction.

Currently, the discrepancy between the experimental and theoretical values of the anomalous magnetic moment of the leptons is tiny³. The loop-quantum correc-

Dirac matrices γ^μ and the electromagnetic field $F_{\mu\nu}$ is

$$\hat{\gamma}^\mu = \gamma^\mu + [p\sqrt{\epsilon} F^\mu{}_\alpha + q\epsilon\phi^\mu{}_\alpha] \gamma^\alpha,$$

where p and q are constants satisfying $2q = p^2 + 1$ and $\epsilon = -1/2\beta^2$. To simplify our exposition here we set $p = 0$.

³ All the values used here were taken from the *Particle Data Group*

tions of the standard magnetic moment make this discrepancy very small, hence restricting the possibility of new physics. The anomalous value a_i of the magnetic moment μ_i of the particle i is defined by

$$a_i = \frac{g_i - 2}{2} = \frac{m_i}{m_e} \frac{\mu_i}{\mu_B} - 1, \quad (12)$$

where $\mu_B = e\hbar/2m_e$ is the Bohr magneton, m_e is the electron mass and g_i and m_i are respectively the Landé factor and the mass of the particle in consideration. For the case of the muon [9], the difference d between the experimental E and the theoretical T values of the anomaly is

$$a_\mu^d \doteq a_\mu^E - a_\mu^T = (1.90 \pm 0.19) \times 10^{-9}. \quad (13)$$

Therefore, this is the upper bound (up to now) in which the geometrical magnetic moment μ_Q could contribute. So, if one considers that effects due to the matter coupling with $\hat{q}^{\mu\nu}$ appear at this order of magnitude, one gets

$$\mu_Q^{(\mu)} \doteq a_\mu^d \frac{e\hbar}{2m_\mu}. \quad (14)$$

On the other hand, our proposal of the geometrical magnetic moment is given by

$$\mu_Q^{(\mu)} = \frac{m_\mu c^2}{\beta}. \quad (15)$$

If one assumes that the geometrical coupling realize all the remaining terms for the muon anomaly, it is possible to estimate from our formula (15) the critical electromagnetic field: $\beta \approx 2.0 \times 10^{23} T$. Assuming that the calibration given by the outcomes above is valid, the geometrical corrections in the case of the magnetic moment of the electron is forecasted to be

$$a_Q^e = 4.44 \times 10^{-14}. \quad (16)$$

The standard experimental (E) and theoretical (T) difference in the case of the electron is about 600 times smaller than the muon [10], that is

$$a_e^d \equiv a_e^E - a_e^T = (3.13 \pm 5.20) \times 10^{-12}. \quad (17)$$

As we can see, the prevailing experimental errors is compatible with \hat{Q} geometry effects in the electron anomaly. Concerning the case of the τ anomaly it is predicted to be less than 13×10^{-3} , which is compatible

with our predictions. Indeed, using our formula (11) we have

$$a_Q^\tau = 1.08 \times 10^{-6}. \quad (18)$$

Let us now discuss the exceptional case of the neutrino. As an uncharged elementary particle, we could expect that neutrino does not have any magnetic moment. Notwithstanding, the standard model predicts a nonzero magnetic moment, which depends linearly on the mass. From our above discussion it seems natural to propose that the existence of the neutrino magnetic moment has only a geometrical origin. Then, we can estimate the value for the neutrino magnetic moment as being given by

$$\mu_\nu = \frac{m_\nu c^2}{\beta} = 8.70 \times 10^{-20} \mu_B. \quad (19)$$

This value is compatible with the range of the values obtained from distinct areas of physics $4 \times 10^{-20} \mu_B < \mu_\nu < 9 \times 10^{-9} \mu_B$ [11], [12] and [13]. Finally we remark that our prediction is close to the established by the Standard Model ($\approx 3.2 \times 10^{-19} \mu_B$).

VIII. CONCLUSIONS

The possibility of describing the dynamics of linear Maxwell theory in terms of the non-linear Born-Infeld theory is certainly an unexpected result. As it was proved in [5], the price to pay is to introduce a curved geometry which has only electromagnetic character. In the present work we try to go one step ahead by generalizing this equivalence in the presence of matter. The geometry in the \hat{Q} representation is associated to all bodies, charged or not. This implies several conditions that we explore here only in the case in which the field is very far from its maximum β . The most unexpected result is the appearance of a magnetic moment for the neutrino. The comparison of our approach to the observational results indicates that our theory is not in contradiction with observation. This model should be further investigated and the case of very high electromagnetic fields examined. We will come back to this elsewhere.

Acknowledgments

We would like to acknowledge J.A. Helayël-Neto, N. Pinto-Neto, F.T. Falciano and E. Goulart for many interesting conversations and the staff of ICRANet in Pescara where this work was done. MN would like to thank FINEP, CNPq and FAPERJ, and EB the CNPq for their financial support.

IX. APPENDIX A: SPINORIAL COVARIANT DERIVATIVE

From the condition (7) it follows that the internal connection $\hat{\Gamma}_\mu$ is provided by

$$\hat{\Gamma}_\mu = \hat{\Gamma}_\mu^{FI} + V_\mu, \quad (20)$$

where V_μ is an arbitrary vector satisfying the Clifford algebra and $\hat{\Gamma}_\mu^{FI}$ is the Fock-Ivanenko connection defined in terms of the $\hat{\gamma}^\mu$'s as follows

$$\hat{\Gamma}_\mu^{FI} = \frac{1}{8} \left[\hat{\gamma}^\lambda \hat{\gamma}_{\lambda,\mu} - \hat{\gamma}_{\lambda,\mu} \hat{\gamma}^\lambda + \hat{\Gamma}_{\lambda\mu}^\epsilon (\hat{\gamma}_\epsilon \hat{\gamma}^\lambda - \hat{\gamma}^\lambda \hat{\gamma}_\epsilon) \right], \quad (21)$$

where $\hat{\Gamma}_{\lambda\mu}^\epsilon$ is the Christoffel symbol constructed with the $\hat{q}^{\mu\nu}$ metric. Thus, the covariant derivative of Ψ is given by

$$\begin{aligned} \hat{\nabla}_\mu \Psi &= \partial_\mu \Psi - \hat{\Gamma}_\mu \Psi \\ &= \partial_\mu \Psi - \hat{\Gamma}_\mu^{FI} \Psi - V_\mu \Psi. \end{aligned} \quad (22)$$

In order to preserve the probability distribution $\hat{\nabla}_\mu (\bar{\Psi} \Psi) = \partial_\mu (\bar{\Psi} \Psi)$, the adjoint of this expression must be

$$\begin{aligned} \hat{\nabla}_\mu \bar{\Psi} &= \partial_\mu \bar{\Psi} + \bar{\Psi} \hat{\Gamma}_\mu \\ &= \partial_\mu \bar{\Psi} + \bar{\Psi} \hat{\Gamma}_\mu^{FI} + \bar{\Psi} V_\mu. \end{aligned} \quad (23)$$

The probability current $\hat{J}^\mu \equiv \bar{\Psi} \hat{\gamma}^\mu \Psi$ is also conserved ($\hat{\nabla}_\mu \hat{J}^\mu = 0$) with the choice of V^μ given by (8), under the condition that $F_{\mu\nu} \hat{q}^{\mu\nu} = 0$.

X. APPENDIX B

The two scalar invariants of the electromagnetic theory are defined as

$$\begin{aligned} F &\equiv F^{\mu\nu} F_{\mu\nu} = 2 \left(|\vec{B}|^2 - |\vec{E}|^2 \right), \\ G &\equiv F^{\mu\nu} F_{\mu\nu}^* = -4 \vec{E} \cdot \vec{B}. \end{aligned}$$

Using these definitions, a direct calculation shows the following algebraic relations

$$F^{\mu\alpha} F_{\alpha\nu}^* - F^{\mu\alpha} F_{\alpha\nu} = \frac{F}{2} \delta^\mu{}_\nu, \quad (24)$$

$$F^{\mu\alpha} F_{\alpha\nu} = -\frac{G}{4} \delta^\mu{}_\nu, \quad (25)$$

$$F^\mu{}_\alpha F^\alpha{}_\beta F^\beta{}_\nu = -\frac{G}{4} F^\mu{}_\nu - \frac{F}{2} F^\mu{}_\nu, \quad (26)$$

$$F^\mu{}_\alpha F^\alpha{}_\beta F^\beta{}_\lambda F^\lambda{}_\nu = \frac{G^2}{16} \delta^\mu{}_\nu - \frac{F}{2} F^\mu{}_\alpha F^\alpha{}_\nu. \quad (27)$$

In Maxwell's theory, every tensor is raised and lowered by the Minkowski metric $\eta_{\mu\nu}$ while in the curved space-time where we defined the Born-Infeld theory must be raised and lowered using $\hat{q}_{\mu\nu}$.

Maxwell Theory: it is given by the $\eta_{\mu\nu}$ -metric and the Lagrangian $L = -\frac{1}{4}F$.

Then,

$$F^{\mu\nu} = F_{\alpha\beta} \eta^{\mu\alpha} \eta^{\nu\beta},$$

$$F = F_{\mu\nu} F_{\alpha\beta} \eta^{\mu\alpha} \eta^{\nu\beta}.$$

Born-Infeld Theory: it corresponds to the $\hat{q}_{\mu\nu}$ -metric and the nonlinear Lagrangian $\hat{L} = \beta^2 \left(1 - \sqrt{\hat{U}} \right)$.

Then,

$$\hat{U} = 1 + \frac{\hat{F}}{2\beta^2} - \frac{\hat{G}}{16\beta^4},$$

$$\hat{F}^{\mu\nu} = F_{\alpha\beta} \hat{q}^{\mu\alpha} \hat{q}^{\nu\beta},$$

$$\hat{F} = F_{\mu\nu} F_{\alpha\beta} \hat{q}^{\mu\alpha} \hat{q}^{\nu\beta},$$

$$\hat{G} = \frac{1}{\sqrt{-\hat{q}}} \eta^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} = \hat{F}^{\mu\nu} F_{\mu\nu}.$$

The $\hat{q}_{\mu\nu}$ metric depends on the electromagnetic field. Due to the algebraic relations (24)-(27), there is a unique way to introduce the electromagnetic metric (EM metric), namely,

$$\hat{q}^{\mu\nu} = a \eta^{\mu\nu} + b \phi^{\mu\nu}, \quad (28)$$

where a and b are two arbitrary functions of the two Lorentz invariants F and G and again $\phi^{\mu\nu} \equiv F^{\mu\alpha} F_{\alpha}{}^\nu$. The term electromagnetic metric is justified by the fact that $\hat{q}_{\mu\nu}$ is uniquely defined in terms of the electromagnetic fields.

-
- [1] W.J. Marciano and A.I. Sanda, *Phys. Lett. B* **67** 303 (1977);
 - [2] B.W. Lee and R.E. Shrock, *Phys. Rev. D* **16** 1444 (1977);
 - [3] H. Belich, L.P. Colatto, T. Costa-Soares, J.A. Helayël-Neto and M.T.D. Orlando, arXiv: hep-ph/0806.1253v2;
 - [4] K. Bhattacharya and P.B. Pal, *Proc. Indian Natn Sci Acad* **70**, **A** 145 (2004);
 - [5] M Novello, F.T Falciano and E. Goulart, *Electromagnetic Geometry*, to be submitted.
 - [6] M. Novello, *Phys. Rev. D*, **8** 8, 2398 (1973);
 - [7] K. Nakamura et al., *Particle Data Group*, *JP G* **37** 075021 (2010);
 - [8] M. Passera, arXiv: hep-ph/0702027v1;
 - [9] A. Czarneck and W.J. Marciano, *Phys. Rev. D* **64** 013014 (2001);
 - [10] D. Hanneke, S. Fogwell, and G. Gabrielse, *Phys. Rev. Lett.* **100** 120801 (2008);
 - [11] A.B. Balantekin, arXiv: hep-ph/0601113v1;
 - [12] O. Lychkovskiy and S. Blinnikov, arXiv: hep-ph/0905.3658v3;
 - [13] A.V. Kuznetsov and N.V. Mikheev *JCAP* **11** 031 (2007).